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A Theory of Brittle Creep in Rock under Uniaxial Compression

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Scholz's theory of brittle creep is rejected. A new theory based on Charles's theory of the subcritical growth of pre-existing cracks in the specimen by stress-aided corrosion is put forward. It is a successful explanation of new experiments on the creep of Pennant sandstone and Carrara marble under uniaxial compression at room temperature.

INTRODUCTION

Many creep experiments on rock under compression were conducted under conditions where the specimen is brittle, that is, it fractures at small strains with loss of cohesion between the fracture surfaces.

Brittleness has certain implications. *Pratt* [1967] has pointed out that, for a material to be able to undergo a general deformation, 'there must be a sufficient number of independent slip systems, distributed homogeneously and able to interpenetrate, with enough mobile dislocations on them to accommodate the applied strain rate.' At least one of these conditions is not fulfilled for most rocks at room temperature.

For the rocks to be capable of a general deformation the component minerals should be deformable. Observed slip systems for rock-forming minerals have recently been compiled by *Handin* [1966] and *Watchman* [1967]. Data on calcite have been added by *Santhanam and Gupta* [1968]. Calcite and quartz are the two minerals that have been most intensively studied, and in neither mineral was there appreciable dislocation mobility at room temperature.

In general, deformation by crystal twinning or slip is insensitive to confining pressure. Only calcite, marble, and halite have been reported to show stress-strain curves insensitive to confining pressure [*Paterson*, 1967]. Deformation in other rocks can be supposed to be cataclastic. The increasing ductility of rocks with increasing confining pressure is due to the inhibiting of fracture propagation [*Pratt*, 1967]. *Murrell* [1965] has shown that the brittle–ductile transition observed in rocks occurred when the stress required to propagate a crack rose to the stress required to overcome sliding friction on the crack. Other features of the stress-strain curves of rocks are also adequately explained on the assumption that the rock is a perfectly elastic body containing an array of pre-existing cracks [*Walsh and Brace*, 1966].

This paper therefore develops a theory of creep in brittle materials based on the assumptions that (1) dislocation motion is negligible, and (2) the material contains pre-existing cracks.

SCHOLZ'S THEORY OF BRITTLE CREEP

It seems that only one theory, that of *Scholz* [1968], has been developed specifically to describe creep in brittle rock. It is reviewed briefly below and is shown to be unsatisfactory.

Scholz suggested that a creep specimen could be considered as a large number of small homogeneous regions (elements) that undergo static fatigue according to equation 1,

$$t_f = (1/a) \exp [(E/KT) + b(F_m - F_a)] \quad (1)$$

where a and b are constants. E is the activation energy of the corrosion reaction that leads to static fatigue. F_m is the instantaneous failure stress of the element, and F_a is the stress on the element causing failure at t_f, the mean fracture time.

Two further assumptions were necessary; as each element fails, it contributes an amount v to the axial strain, and each region acts

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independently and only fails once. Scholz derived the theory for volumetric strains but he commented [*Scholz*, 1968, p. 3299], 'v can also be considered to be the increment of axial or lateral strain.'

If $P(F_a)$, the transitional probability of fracture at a stress F_a, does not vary with time, the probability P that an element will fracture in the next time interval dt after a time t under stress F_a is given by

$$P = P(F_a) \exp\left[-P(F_a)t\right] dt \qquad (2)$$

and this leads to

$$P(F_a) = 1/t_f \qquad (3)$$

Substituting equation 3 in equation 1 gives

$$P(F_a) = a \exp\left[-(E/KT) - b(F_m - F_a)\right] \quad (4)$$

If $N(F_a, t)$ is the number of elements under stress F_a at time t, then the probability of one of these elements failing in the subsequent time interval dt is

$$f(F_a) = N(F_a, t)P(F_a)\, dt \qquad (5)$$

$$d[N(F_a, t)]/dt = N(F_a, t)P(F_a) \qquad (6)$$

The axial creep rate is then

$$\dot{e}_t = v \int_0^{F_m} N(F_a, t)P(F_a)\, dF_a \qquad (7)$$

Integration of equation 6 from zero to time t gives

$$N(F_a, t) = N(F_a, 0) \exp\left[-P(F_a)t\right] \qquad (8)$$

Differentiation of equation 4 leads to

$$d(P(F_a)) = -b\, P(F_a)\, dF_a \qquad (9)$$

Assuming that the initial distribution, $N(F_a, 0)$ is uniform in the interval zero to F_m, and is zero outside it, then $N(F_a, 0) = N$. Equation 7 can then be written

$$\dot{e}_t = vN \int_0^{F_m} P(F_a) \exp\left[-P(F_a)t\right] dF_a$$
$$= (vN/b) \int_0^{F_m} \exp\left[-P(F_a)t\, d(P(F_a))\right] \qquad (10)$$

The integration of equation 10 leads to

$$\dot{e}_t = vN/bt \qquad (11)$$

Scholz's contribution, based on the assumption represented by equation 1, comprises two statements:

$$t_f = c \exp\left[b(F_m - F_a)\right] \qquad (12)$$

$$t_f = d \exp(E/KT) \qquad (13)$$

Equation 12 described the static fatigue of the elements at constant temperature; equation 13 described their static fatigue at constant stress. Scholz suggested that equation 12 could be verified by experiments on the static fatigue of homogeneous specimens of silicates such as glass.

Criticism of Scholz's Theory

Scholz, then, has assumed that a creep specimen is composed of a number of elements of the same dimensions and with similar physical and chemical properties (that is, they all obey the same law of statc fatigue). The stress distribution in each element is assumed to be uniform, and the elements are each stressed to different stresses in the range from zero to the instantaneous compressive-strength of an element. Under compression of the specimen, tensile stresses are assumed to be absent.

There are immediate difficulties with these assumptions. One of these is the definition of the instantaneous compressive-strength of an element. Fracture of bodies under compression is invariably attributed to tensile stresses at cracks and other stress concentrations within the body. *Scholz* [1968, p. 3298] was clear, however, that there are no tensile stresses within the specimen; it is therefore difficult to envisage the occurrence of a fracture.

Notice, also, that the stress distribution within the specimen is specialized. If the stress distribution within the elements is uniform, then their boundaries will be free of shearing stresses, for instance. Scholz has not discussed what arrangement of the elements would produce this stress distribution. However, if the elements are to have perfectly smooth margins to eliminate shearing stresses, then the specimen may not cohere.

Scholz's theory can also be criticized for the form of equation 12. Taking logarithms of equation 12,

$$\log t_f = \log c + b(F_m - F_a) \qquad (14)$$

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From equation 14, a plot of the logarithm of the time to failure of the fatigue specimen against the applied stress should therefore be linear.

The three main groups of data that Scholz quoted, *Charles* [1959], *Mould and Southwick* [1959], and *Glathart and Preston* [1946], were collected to determine the relationship between F_a and t_f. All these authors displayed the data on $F_a - \log t_f$ plots. To connect data collected under similar environmental conditions, they drew best-fit curves, not straight lines, through the data. The curves were generally concave upwards.

Glathart and Preston [1946, p. 189] explicitly rejected equation 12: 'Baker [*Baker and Preston*, 1946] adopted the rather natural method of plotting (F_a against $\log t_f$) and obtained very definitely curved-lines, the curvature being more obvious because of his longer range of time intervals.' They reported that the data were adequately explained by equation 15

$$\log t_f = -a + b/F_a \qquad (15)$$

Mould and Southwick [1959] considered four proposed static-fatigue laws to explain their data and that of *Glathart and Preston* [1946]. In addition to equation 15, they tried equations 16, 17, and 18.

$$\log t_f = a - (b/F_a) - \log F_a \qquad (16)$$

which was suggested by *Stuart and Anderson* [1953],

$$\log t_f = -a + (b/F_a^2) \qquad (17)$$

from the work of *Elliott* [1958], and

$$\log t_f = -a - b \log F_a \qquad (18)$$

where a and b are positive constants (though not the same constants in each equation). Equations 15 to 18 are predicted by various models of the corrosion process at the crack tip.

Equation 18 was the only static-fatigue law 'in complete agreement with the data obtained in the study' [*Mould and Southwick*, 1959, p. 591].

Charles [1958] reported that his data were well fitted by equation 18.

Unfortunately the full experimental data have not been published by any of the authors, and the graphical representations are too small to describe the data accurately. Charles conducted tests on groups of soda-glass specimens at the same pre-set stress. He then selected the mode of the logarithm of the time to failure, and plotted it against the logarithm of the stress.

It is doubtful whether the stress in the other two groups of experiments was sufficiently closely controlled to allow it to be treated as an independent variable. Notice, also, that 'averages' of the times to failure of the groups of specimens were plotted. Because the averages were unidentified, it is probable that they are arithmetic averages of the times to failure. The form equation 18 would require that the arithmetic averages of the logarithms of the times to failure be plotted against the logarithm of the stress.

Thus the fit of various functions to the static-fatigue data remains a matter of opinion, but the weight of evidence seems to favor equation 18 over equation 12. Charles's theory of static fatigue might form a more satisfactory basis for a theory of brittle creep than that of Scholz [*Charles*, 1958].

CHARLES'S THEORY OF STATIC FATIGUE

To provide background for this theory, it will be necessary to review very briefly the data on static fatigue of silicates, the principal rock-forming material.

Charles and Gurney and Pearson demonstrated that static fatigue in glass was negligible in a vacuum. It has also been shown that vacuums reduce the effects of static fatigue on basalt [*Krokosky and Husak*, 1968], on ceramics [*Baker and Preston*, 1946], on sintered alumina [*Pearson*, 1956], and on fused silica-rods [*Le Roux*, 1965; *Hammond and Revitz*, 1963]. *Charles* [1958], *Schoening* [1960], and *Gurney and Pearson* [1949] demonstrated that static fatigue of glass was accelerated by high concentrations of water vapor. *Le Roux* [1965] demonstrated the same effect of water vapor in the fatigue of fused silica; Gurney and Pearson showed that the presence of carbon dioxide in the surrounding environment accelerated fatigue of glass. These studies show that the fatigue of a wide range of brittle materials is dependent on the ambient environment.

The common hypothesis of these experiments was that static fatigue is due to stress-aided corrosion at the tips of microcracks in the specimen. This causes the cracks to lengthen. After a period of time under a sustained tension, a crack reaches a critical length [*Jaeger*, 1962, p. 85] and propagates unstably, causing fracture of the specimen.

Charles [1958] considered a highly elliptical hole of major axis L in a flat glass plate subject to an average tensile-stress S_y in a direction perpendicular to the major axis of the crack. He suggested

$$\dot{L}_x = f(S_x) + k \tag{19}$$

where $\dot{L}_x$ is the velocity of the crack in the x direction. S_x is the tensile stress at the tip of the crack tangential to the crack surface; k is the corrosion rate at zero tangential stress. Suppose

$$f(S_x) = c(S_x/S_{cr})^n \tag{20}$$

where n is a positive constant, c is the maximum velocity of the crack, and S_{cr} is the tensile strength of the atomic bonds at the crack tip. As

$$S_x/S_y = 2(L/r)^{1/2} \tag{21}$$

$$S_{cr}/S_y = 2(L_{cr}/r)^{1/2} \tag{22}$$

where r is the curvature at the crack tip, L_{cr} is the length at which occurs the critical stress S_{cr} for rupture of bonds at the crack tip. Equations 21 and 22 are derived from the theory of stress concentrations around holes in perfectly elastic bodies [*Jaeger*, 1962, p. 85].

Substituting equations 20 and 21 in equation 19 gives equation 23,

$$\dot{L}_x = c(L/L_{cr})^{n/2} + k \tag{23}$$

Charles suggested that, if static fatigue is to take place, stress-activated corrosion must occur at a much greater rate than stress-free corrosion. Hence the crack would grow with constant curvature r until it reaches its critical length L_{cr}. If stress-free corrosion were as important as stress-activated corrosion, the crack growth would occur with increasing radius of curvature, and the stress concentration might be seriously reduced.

The corrosion rate at zero stress k can therefore be neglected in equation 23 by comparison with the stress-dependent corrosion rate.

The temperature dependence of crack growth can be introduced by the assumption that corrosion is a rate process with an activation energy A. The experimentally determined activation energy of the process below 150°C is close to that for the diffusion of sodium atoms in glass. Charles suggested that the sodium atoms catalyse the hydrolysis of the oxygen-silicon bond in glass by creating free hydroxyl ions. Equation 23 can be written

$$\dot{L}_x = B(L/L_{cr})^{n/2} \exp(-A/KT) \tag{24}$$

Integrating equation 24 with respect to time gave equation 25,

$$\int_{L_0}^{L_{cr}} dL(L/L_{cr})^{-n/2} = \int_0^{t_f} B \exp(-A/KT)\, dt$$

$$[2L_{cr}/(n-2)][(L_{cr}/L_0)^{(n-2)/2} - 1] = B \exp(-A/KT)t_f \tag{25}$$

When n and t_f are large, equation 25 can be written,

$$[2L_{cr}/B(n-2)] \cdot \exp(A/KT)(L_{cr}/L_0)^{(n-2)/2} = t_f \tag{26}$$

Taking logarithms of equation 26,

$$\log t_f = (n/2) \log L_{cr} - \log D \tag{27}$$

where

$$D = L_0^{(n-2)/2}(B/2)(n-2) \exp(A/KT)$$

Equation 22 can be rewritten as equation 28,

$$L_{cr} = rS_{cr}^2/4S_y^2 \tag{28}$$

Substituting equation 28 in equation 27,

$$\log t_f = -n \log S_y - \log D' \tag{29}$$

where

$$D' = (rS_{cr}^2/4)^{-n/2} \cdot D$$

Equation 29 gave the static-fatigue law (equation 18). The parameter n can be determined from the slope of a $\log t_f - \log S_y$ plot. *Charles* [1958] reported a value of about 16.

The growth of subcritical cracks under tension has now been directly observed in glass microscope slides [*Wiederhorn*, 1967] and in sapphire [*Wiederhorn*, 1968].

It was found that the growth of a crack can be divided into two stages: a stage where crack

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motion is relatively slow, and a stage of catastrophic motion initiated when the crack is long enough to satisfy the Griffith criterion for crack initiation.

The time dependence of static fatigue is controlled by crack growth in the first region. The time taken to traverse the other region is negligible by comparison.

Wiederhorn's data on the stress dependence of crack growth in glass in the first region can be replotted on double logarithmic coordinates, $\log \dot{L} - \log P$, which is equivalent to assuming a stress dependence of the form $\dot{L} = CP^n$. Taking logarithms,

$$\log \dot{L} = \log C + n \log P$$

The data seem to be adequately explained by this relationship. Indeed, compared to the exponential relationship $\dot{L} = A \exp(BP)$ suggested by *Wiederhorn* [1968], the scatter of the data is reduced. A more detailed analysis cannot be justified since numerical values of $\dot{L}$ and P were not given and P has a small range. The parameter n can be determined approximately from the slope of the fitted line and has a value of about 19.

The agreement between Wiederhorn's data and that of *Charles* [1958] for the value of n for glass is reasonable, particularly as they were working on different glasses. Therefore the main assumption of Charles's theory is plausible. There seems little point in using the more sophisticated versions of Charles's model for glass under tension [*Wiederhorn*, 1967] while there is no experimental work on subcritical crack growth in glass under compression.

The Effect of Uniaxial Compression

The extension of Charles's theory to the growth of cracks under uniaxial compression involves some problems with the stresses at the crack margins.

The Griffith criterion for the initiation of crack propagation does not predict the behavior of a propagating crack. *Wells and Post* [1958] have shown that a propagating crack under uniaxial tension in a direction normal to its direction of propagation will extend its own plane to a surface boundary. This result has been confirmed experimentally by *Brace and Bombolakis* [1963] and by *Hoek* [1965] for cracks in glass sheet.

Because all Charles's experiments on static fatigue were performed on specimens under bending or uniaxial tension, his model of the process was adequate to describe his results.

Hoek has confirmed empirically the Griffith criterion for fracture initiation from open cracks in glass plates in uniaxial compression. In a modified form, to allow for friction between the crack surfaces, the criterion also applies to closed cracks. The behavior of the propagating crack in compression is much more complex than in tension.

Brace and Bombolakis [1963] reported experiments on open cracks in glass plates under uniaxial compression. At the critical stress, branch fractures propagated from the ends of the cracks and 'became nearly parallel with the direction of compression . . . . when this direction was attained further crack growth stopped, apparently because of the decrease of tensile stress-concentration at the tip of the crack . . . . considerable increase in compressive stress is required to cause additional growth of these cracks.' *Brace, Paulding, and Scholz* [1966] refer to this sequence of events as 'crack hardening.'

Presumably, the effect of tensile stress at the crack tips at the atomic level is to stretch the bonds between the atoms allowing easier passage to diffusing sodium ions and hence increasing corrosion rates. Compressive stress at the crack tips will inhibit corrosion, and the stress-dependent corrosion rate for compressive stresses may be less than the corrosion rate at zero stress. Under these conditions, crack growth may result in the elimination of the stress concentration.

Jaeger has reviewed the general problem of stress concentrations around holes in a perfectly elastic medium.

Consider an elliptical hole of major axis a and minor axis b in an infinite, perfectly elastic plane. The solution [*Jaeger*, 1962, pp. 198–199] for the stresses used elliptical coordinates, $x = \cosh z \cos u$, $y = c \sinh z \sin u$; then $a = c \cosh x$, $b = c \sinh z$, and the hole is bounded by $z = z_0$. The plane is subject to stress P_1 at infinity inclined to the x axis (which lies along the major axis of the ellipse) at an angle α.

The tangential stress S_r at the boundary of the hole is, according to *Jaeger* [1962, p. 199, eq. 29],

$$S_x = \frac{P_1[2ab + (a^2 - b^2) \cos 2\alpha - (a + b)^2 \cos 2(\alpha - u)]}{a^2 + b^2 - (a^2 - b^2) \cos 2u} \quad (30)$$

tensile stresses are negative.

The stress at the crack tip is given by equation 30 with $u = 0$. For a flat crack, a is much greater than b. Then, equation 30 can be written as equation 31,

$$S_x = (P_1/b)(a - (b + a) \cos 2\alpha) \quad (31)$$

Equation 30 shows that if P_1 is tensile, the stress at the crack tip is tensile except where α is very close to zero; that is, when the major axis of the crack is nearly parallel to the applied tension. If P_1 is compressive, S_x is compressive except when α is very close to zero. The maximum value of the tensile stress at the crack tip is P_1 $(1 + 2a/b)$ when P_1 is tensile $(\alpha = 90°)$ and P_1 when P_1 is compressive $(\alpha = 0)$.

When P_1 is compressive and $\alpha = 0$, notice that the tensile stress at the crack tip is independent of the form of the crack. Unless P_1 approaches the tensile strength of the atomic bonds at the crack tip, the crack cannot propagate catastrophically.

Hoek [1965, p. 16] pointed out that, while the maximum tensile stress tangential to the crack surface of flat cracks occurred near the crack tip, it did not occur at the crack tip. He simplified equation 30 by assuming that u is small, and b is small compared to a [*Hoek*, 1965, appendix 1]. By differentiating the resulting expression with respect to u, Hoek was able to show that the maximum tensile stress S_t near the crack tip is given by

$$S_t z_0 = P_1(\sin^2 \alpha \pm \sin \alpha) \quad z_0 = b/2a \quad (32)$$

When P_1 is compressive, the negative sign in equation 32 is appropriate; S_t will always be negative (tensile) when P_1 is compressive, except when $\sin \alpha = 1$ or 0, then S_t is indeterminate. Notice that, as the positive sign in equation 32 should be used when P_1 is tensile, S_t is always tensile and considerably larger than its value when P_1 is compressive.

Hoek's approximation leads to errors when α is close to zero or 90°. This can be seen by comparing equation 32 with equation 31, (which is exact) or from the predicted positions of S_t. These are given by equation 33 [*Hoek*, 1965, appendix 1].

$$V_t = -b/2a(\tan \alpha \pm \sec \alpha) \quad (33)$$

The errors arise because some products of b and trignometric functions of α removed by the simplification of equation 30 are not negligible when the trigonometric functions take extreme values.

A more elaborate analysis than Hoek's is required to determine the exact situation. It will not be attempted here. Instead, notice that symmetry considerations suggest that the maximum tensile stress is at the crack tip when the crack major axis is parallel or perpendicular to the principal stress, and that equation 33 suggests that, in other positions, the maximum tensile stress is at some distance from the crack tip.

The situation is more complex when the crack is closed. *Hoek* [1965, p. 24] used the same approximations as he made in the case of open cracks to show that on McClintock and Walsh's hypothesis of the behavior of closed cracks,

$$S_t z_0 = P_1 \sin (\cos \alpha - m \sin \alpha) \quad (34)$$

where m is the coefficient of friction on the crack surface. The stress S_t is tensile for values of $\cos \alpha$ greater than $m \sin \alpha$. Taking m to be equal to one, closed cracks inclined at more than 45° to P_1 will not, then, grow in uniaxial compression.

A New Theory of Brittle Creep

We now use this discussion of stress distribution around cracks and Charles's theory to explain brittle creep in uniaxial compression.

Suppose that a subcritical crack in uniaxial compression extends in its own plane by stress corrosion due to the tensile stress near the crack tips, and that when it reaches a critical length, it propagates in the manner described by *Brace and Bombolakis* [1963].

This sequence may seem less plausible than assuming that the crack grows along the path of a hypothetical branch fracture. However, the alternative leads the crack to a stable configura-

$$+ b)^2 \cos 2(\alpha - u)] \over \cos 2u \quad (30)$$

e given by equation 33 [*Hoek*, 1965, 1].

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sequence may seem less plausible than g that the crack grows along the path othetical branch fracture. However, the ive leads the crack to a stable configura-

tion without giving rise to any event that could cause the microseismic emission commonly observed in brittle creep. The principal contribution to creep strain comes from strains and displacements about propagating cracks. Once these cracks have propagated, they are stable or 'crack hardened.'

Sack [1946] has shown that results for stresses around flat cracks in two dimensions can be extended naturally to three dimensions, to flat cracks with a circular plan. These cracks have been termed 'penny-shaped' cracks. The maximum tensile stress on the crack margin lies in the plane of the minimum and maximum principal stresses and differs only by a constant from the value predicted by equation 30 for cracks in two dimensions.

Suppose there are $M(L, \alpha)\, dL$ cracks in the creep specimen with lengths at zero time between L_0 and $L_0 + dL$ at angles to the principal stress between α and $\alpha + d\alpha$. If each crack caused a strain increment v on propagating, the total strain de due to those cracks is $M(L, \alpha)\, v dL$. The time t_f for a crack length L_0 to grow to its critical length L_{cr} is given from equation 25 by

$$t_f = \exp(A/KT) L_{cr}^{\,n/2} \cdot [2/B(n-2)] L_0^{-[(n-2)/2]} \quad (35)$$

$$t_f = E L_0^{-[(n-2)/2]}$$

defining E.

Similarly, the time $(t_f - dt)$ for a crack of length $(L_0 + dL)$ to grow to L_{cr} is given by

$$(t_f - dt) = E(L_0 + dL)^{-[(n-2)/2]} \quad (36)$$

Substracting equation 36 from equation 35,

$$dt = E L_0^{-[(n-2)/2]} \cdot [1 - (1 + dL/L_0)]^{-[(n-2)/2]} \quad (37)$$

If dt and dL are small and n is large, equation 37 can be written

$$dt = [(n-2)/2] E L_0^{-n/2}\, dL \quad (38)$$

Then the strain rate at t_f due to the propagating cracks is $de/dt_f = M(L, \alpha)\; v\; dL/dt$,

$$de/dt_f = [2/E(n-2)] L_0^{\,n/2} \cdot M(L, \alpha) v \quad (39)$$

It would be reasonable to expect more short cracks than long ones. Thus $M(L, \alpha)$ is unlikely to be independent of L_0. Unfortunately, there is no direct way to determine the distribution of crack lengths.

Gilvarry [1961] suggested the basis of an indirect method. He considered the size distribution of the fragments in the single fracture of an infinitely extensive brittle-body due to the propagation of internal flaws. He divided internal flaws into three types, depending on the number of flaws against which they terminate. There are volume, facial, and edge flaws terminating against zero, one, or two flaws, respectively. Further classes were excluded because many of the fragments at fracture were four-sided. Gilvarry found

$$g = 1 - \exp[-(x/k) - (x/j)^2 - (x/i)^3]$$

where g is the volume (or weight) passing a mesh size x; k, j, i are the average spacings of edge, facial, and volume flaws. If x is small, then this may be written,

$$g = 1 - \exp[-(x/k)]$$

and if x is very small,

$$g = (x/k) \quad (40)$$

so that fragments passing the smallest mesh size are controlled by edge flaws.

The weight dg in a size interval dx is given by differentiating equation 40,

$$dg/dx = 1/k \quad (41)$$

As $dg = Nk'L^3$ where N is the number of fragments in size interval, the number of fragments with average size L is given by

$$N = k'' L^{-3} \quad (42)$$

Equation 42 has been confirmed experimentally by a number of workers [*Gilvarry and Bergstrom*, 1961]. In particular, *Hamilton and Knight* [1958] report the exponent of L to be about -2.75 for Pennant sandstone.

Single fracture has been defined by *Gilvarry* [1961] as 'fracture by an external stress system which is instantly and permanently removed when the first one or few Griffith cracks begin to propagate. Subsequent flaws are activated by stress waves produced by propagation of prior ones . . . . ' Assume, then, that the smallest fragments are bounded by flaws close to their original lengths so that the length distribution of the flaws can be written,

$$M(\alpha)L^{-m} = M(L, \alpha) \quad (43)$$

Substituting equation 43 in equation 39 gave equation 44,

$$de/dt_f = [2/(n-2)E]L_0^{(n-2m)/2}M(\alpha)v \quad (44)$$

Since $L_0 = (t_f/E)^{-[2/(n-2)]}$ from equation 35, equation 44 can be written

$$de/dt_f = [2/(n-2)] \cdot E^{-2(m-1)/(n-2)}t^{-(n-2m)/(n-2)} \cdot M(\alpha)v \quad (45)$$

$$de/dt_f = \{[B \exp(-A/KT)]^{(2m-2)} \cdot [2/(n-2)t]^{n-2m}L_{cr}^{-n(m-1)}\}^{1/(n-2)}M(\alpha)v \quad (46)$$

From equation 32,

$$S_{cr} = S_y \cdot 2(L_{cr}/r)^{1/2}(\sin^2\alpha - \sin\alpha) \quad (47)$$

using Charles's terminology for cracks inclined at y to the principal compressive-stress S_y. Substituting equation 47 into equation 46 gives equation 48

$$de/dt_f = \{[B \exp(-A/KT)]^{(2m-2)}[2/(n-2)t]^{n-2m} \cdot [S_y(\sin^2\alpha - \sin\alpha)/S_{cr} \cdot r^{1/2}]^{2n(m-1)}\}^{1/(n-2)}M(y)v \quad (48)$$

The creep rate of the whole specimen is the sum of the values of equation 48 over all the appropriate values of α. Equations 32 and 47 are inaccurate when α is near zero or ninety degrees. Cracks at very high or very low angles to the compressive stress will make only a small contribution to the total strain since the tensile stresses at their tips are comparatively small. There is, then, probably no serious error in evaluating the sum only between the limits of, say, eighty-five and five degrees and equation 48 is exact when all the cracks lie within one plane.

It is not possible to predict the value of the creep rate from equation 48 because there are considerable uncertainties in the values of A, B, v, and $M(\alpha)$. However, as equation 48 predicts the time, temperature, and stress dependence of transient-creep rate in the specimen, the theory can still be tested.

The Analysis of Some New Creep Experiments

In what follows, some new creep experiments on rock under uniaxial compression at room temperature are analyzed by the structural theory.

Equation 48 can be written

$$de/dt = KS_y^{2n(m-1)/(n-2)}t^{-(n-2m)/(n-2)}$$

where K is a constant, or as

$$de/dt = b_1 t^{b_2} \quad (49)$$

where $b_1 = KS_y^{2n(m-1)/(n-2)}$, $b_2 = -(n-2m)/(n-2)$. Then b_1 is the strain rate at unit time, and b_2 is a strain-hardening parameter measuring the rate of decrease of the strain rate with time.

Notice that in the trivial case where m is exactly one, b_2 is minus one; this leads to a logarithmic creep law [*Scholz*, 1968]. Another consequence is that the creep rate is independent of the stress. Equation 48 also shows that, when m is close to minus one, small changes in m will cause large changes in the stress dependence of the creep rate; the time dependence is, however, much less sensitive. This emphasizes the special nature of the logarithmic creep law, $(de/dt) = b_1 t^{-1}$, which is transitional between creep laws of the form

$$e_t - e_0 = [b_1/(b_2+1)]t^{b_2+1} \qquad b_2 > -1$$

where there is no limit to the amount of transient creep with time, and the form

$$e_t - e_1 = [-b_1/(b_2+1)](1 - t^{b_2+1}) \qquad b_2 < -1$$

where creep tends to a finite limit with time. e_0, e_1 are the creep strains at zero and one time unit.

Changes in the value of m with stress are not implausible in the structural theory, but they lead to complications. As two experiments at different stresses are required to calculate a value of m, and at least three are required to test the power-law dependence of strain rate on stress, n and m cannot be determined if m changes rapidly with stress.

If the strain-hardening parameter b_2 is constant over a range of stresses, n and m can be are analyzed by the structural

48 can be written

$= KS_y^{2n(m-1)/(n-2)} t^{-(n-2m)/(n-2)}$

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$$S_{cr} \cdot r^{1/2}]^{2n(m-1)}\}^{1/(n-2)} M(y)v \qquad (48)$$

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estimated from the exponent of the power-law dependence of strain rate on stress, and from the mean value of b_2. *Cruden* [1969] reported a series of creep experiments on Carrara marble and Pennant sandstone.

The parameters of the fit of the law [equation 49] to these experiments are tabulated as a function of the percentage of the short-term failure stress in the uniaxial compression P_s at which the experiment was conducted (Table 1). The fit was performed by fitting the straight line $\log (de/dt) = \log b_1 + b_2 \log t$ by simple linear regression on the assumption that the times at which the strain rates are measured are without error.

All the experiments are satisfactorily described by the power law of creep above (equation 49) [*Cruden*, 1969]. *Scholz* [1968] has suggested that the true value of b_2 is -1. But none of the experiments in Table 1 have estimates of b_2 that are exactly -1. In two of the experiments (on Pennant sandstone at 65% P_s and on Carrara marble at 53% P_s), the possibility that the true value of b_2 is -1 can be rejected at the 1% confidence level.

In Figure 1, b_2 for these experiments is plotted against stress. The strain-hardening parameters of Pennant sandstone do not appear to be stress dependent, but there is a significant decrease of b_2 for Carrara marble below 70% of the failure stress. Unfortunately, the experiment at 53% P_s showed only 10 microstrains creep in eight days, and experiments at lower stresses would have been beyond the accuracy of the apparatus used.

In Figure 2, the logarithms of the strain rates in the experiments at a time h are plotted against the stress (on a logarithmic scale). Strain rates are, of course, most precisely determined by the regression at the mean of the logarithms of the times of the observations [*Hald*, 1952]. The time h is the weighted mean of the means of the logarithms of the estimated times of observation of the strain rates in the experiments.

If the strain rates followed a power-law dependence on stress, the data would plot on a straight line in Figure 2. The Pennant sandstone data fall on at least two separate straight lines, one from experiments at 35% P_s and below, and one for those above this value.

Because the strain-hardening parameters of the sandstone experiments are not significantly different, the weighted mean of the values of

TABLE 1. Parameters of Fit of Creep Law $de/dt = b_1 t b_2$ to Experiments on Pennant Sandstone and Carrara Marble

	$\%P_s$	$\log b_1$	b_2	dw	R_1	w	σb_2	σb_1
Pennant sandstone								
	15	1.77	−0.91	1.80	719.1	40	0.034	0.20
	25	2.15	−0.93	1.95	250.5	13	0.058	0.26
	35	2.48	−0.97	1.50	269.3	30	0.059	0.26
	45	2.61	−1.01	1.31	77.7	11	0.050	0.11
	50	2.42	−0.91	2.57	478.2	28	0.042	0.18
	65	2.50	−0.86	2.09	607.0	29	0.035	0.18
	75	3.27	−0.98	2.18	687.8	31	0.038	0.18
	85	3.12	−0.94	2.21	180.3	25	0.070	0.35
Carrara marble								
	53	4.77	−2.11	3.03	256.9	15	0.13	0.80
	64	1.67	−1.22	1.91	367.2	24	0.063	0.29
	70	0.31	−0.79	1.85	34.7	8	0.081	0.33
	77	2.11	−1.11	1.98	149.3	48	0.091	0.40
	83	2.21	−1.02	1.46	415.4	44	0.050	0.23
	86	0.77	−0.87	2.51	51.9	9	0.098	0.59

Notes.

$\log b_1$ is the natural logarithm of b_1 (b_1 is measured in microstrains per minute).
dw is the Durbin-Watson statistic [*Durbin and Watson*, 1951].
w is the weighting of the regression parameters.
σb_1 and σb_2 are the standard deviations of b_1 and b_2.

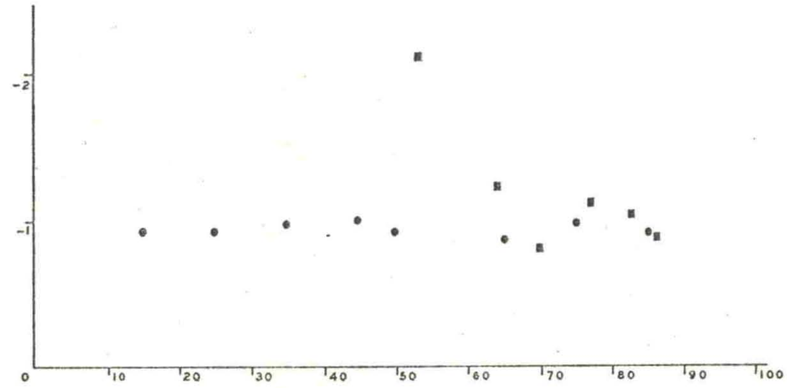


Fig. 1. Plot of the value of the strain-hardening parameter b_2 (vertical axis) against the percentage of short-term failure stress P_s (horizontal axis). Circles indicate Pennant sandstone; squares indicate Carrara marble.

b_2 for the experiments at loads of 3.5 tons and less can be used to write, from the structural theory,

$$(n - 2m)/(n - 2) = 0.930$$

The exponent of the power-law dependence of strain rate on stress can be determined by regressing the logarithms of the strain rates against the logarithms of the stresses. Then,

$$2n(m - 1)/(n - 2) = 0.58$$

These equations can be uniquely solved for n and m since the root $n = 2$ can always be discarded on physical grounds. Solution gave $n = 8.3$, $m = 1.22$; these values are in the ranges suggested by the theory.

Assuming that n, which measures the increase in corrosion rate caused by stretching the mineral lattice, is a constant of the mineral and is not stress dependent, values of m can be calculated for higher loads. They are listed below.

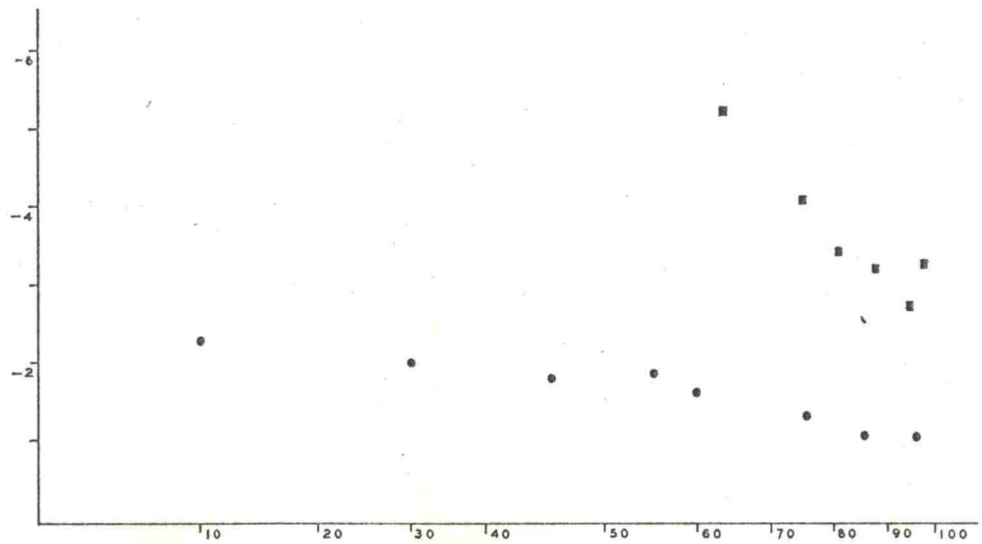
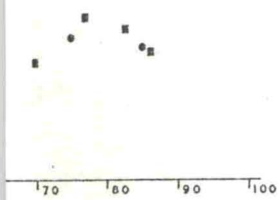


Fig. 2. Plot of the natural logarithm of the strain rate (vertical axis), in microstrains per minute, against the percentage of short-term failure stress (horizonal axis) logarithmic scale. Circles indicate Pennant sandstone; squares indicate Carrara marble.



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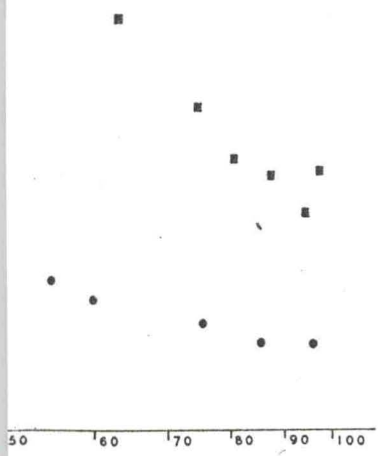

: (vertical axis), in microstrains per
s (horizonal axis) logarithmic scale.
ra marble.

$\%P_s$	m	$2n(m-1)/(n-2)$
45	0.97	-0.079
50	1.24	0.63
65	1.44	1.16
75	1.06	0.16
85	1.22	0.58

Since m is not a constant above 35% of the short-term failure load, the value of $2n(m-1)/(n-2)$ does not represent the exponent of the power-law dependence of strain rate on stress. A rough value of this exponent is about 1.2 [*Cruden*, 1969]. Therefore, at about a third of the failure strength, the exponent approximately doubles. *Evans* [1958, p. 182] reported a similar phenomenon in creep experiments on concrete.

The data on the creep of Carrara marble are complicated by the stress dependence of the strain-hardening parameter b_2.

Another problem is the value of b_2 from the Carrara marble experiment at 53% of the failure load. From this,

$$(n-2m)/(n-2) = 2.11$$

Inspection shows that if $m = 0$ and $n = 10$ the value of the right-hand side is 1.25. The lowest reasonable estimate of b_2 for this experiment is about -1.8. Thus, either n must be about four with m zero, or m must be negative.

Consider the possibility that m is negative. This implies that the number of cracks increases with the length of the crack. At loads above 64% of the failure load, where shorter cracks will be making their contribution to creep, there is no need to suppose that m is negative, and the number of cracks can then be supposed to decrease with their length. Thus the crack-length distribution has a maximum grouped around the cracks that propagate early in transient-creep experiments at about 64% of the failure load.

If the experiments on Carrara marble at 64% of the failure load and below are omitted from the analysis, b_2 can reasonably be supposed to be constant. The reduced body of data can, again, be examined by regressing logarithms of the strain rates against the logarithms of the loads. The results are

$$2n(m-1)/(n-2) = 2.36$$

$$(n-2m)/(n-2) = 0.976$$

The equations lead to estimates of n as 98.5 and m as 2.16. The estimate of n is large. However, little confidence can be placed in the mean value of the strain-hardening parameter b_2; lower values of b_2 would lead to considerably lower estimates of n.

Conclusions

The creep data thus show distinctly different patterns of behavior for Pennant sandstone and Carrara marble. For Pennant sandstone, the value of b_2 is just greater than minus one, and the stress dependence of the strain rates is linear to a crude approximation. Carrara marble shows a stress-dependent, strain-hardening parameter, and the strain rates are roughly proportional to the square of the stress.

The structural theory attributes the difference in behavior to differing corrosion reactions in silicates and carbonates resulting in different values of n and to differing crack-length distributions. The length distribution of cracks in the two rock types can be derived from the calculated values of m.

The most pronounced difference between the two distributions is the relative deficiency of the marble in long and short cracks; the crack-length distribution has a maximum. *Brace* [1964, p. 153] suggested that the maximum crack length in a rock sample was a function of the grain-size distribution. Thus the clustering of the size distribution of the cracks about a broad maximum would appear a consequence of the equigranular texture of the marble described by *Ramez and Murrell* [1964].

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